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On unique predictions for single spin azimuthal asymmetry

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Abstract. Theoretically there are two approaches to predict single spin azimuthal asymmetries. One is to take transverse momenta of the partons into account by using transverse momentum dependent parton distributions, while another is to take asymmetries as twist-3 effects. The non-perturbative effects in these approaches are parameterized with different matrix elements, and predictions can be different. Recently, gauge invariant definitions of transverse momentum dependent parton distributions were derived. With these definitions it can be shown that there are relations between non-perturbative matrix elements in these two approaches. These relations may enable us to unify two approaches and to have unique predictions for single spin azimuthal asymmetries. In this letter we derive these relations by using time-reversal symmetry and show that even with these relations the single spin azimuthal asymmetry in a Drell–Yan process is predicted differently in different approaches.

Single spin azimuthal asymmetry provides a new tool for the study of the structure of hadrons because the asymmetry is sensitive to correlations between quarks and gluons as partons inside a hadron and to orbital angular momenta of these partons. Experimentally, such an asymmetry was observed for inclusive production of pions in polarized proton–antiproton scattering with center-of-mass energy $\sqrt{s} = 20\,\mathrm{GeV}$ by the E704 collaboration [1]. The asymmetry is large for a charged pion, while for π^0 production it is consistent with zero when the transverse momentum k_T is smaller than 3 GeV, and it tends to a positive value when k_T becomes large. In semi-inclusive deep-inelastic scattering (SIDIS) significant asymmetries were also observed in the production of pions and kaons by HERMES [2]. Asymmetries in polarized proton scattering are currently studied by STAR at RHIC. Large spin effects are observed in the preliminary results after a first run. SIDIS measurements of asymmetries with a transversely polarized target were reported by the SMC collaboration [3]. Experiments with a transversely polarized target are now being performed by HERMES and COMPASS [4,5].

Single spin azimuthal asymmetry is a T-odd effect and helicity-flip amplitudes are involved. Perturbatively T-odd effects can be generated at loop level in hard scattering of active partons of the hadrons. Because the quark–gluon coupling of QCD conserves helicities in the massless limit, the T-odd effects are proportional to quark masses which can be neglected. Therefore the observed T-odd effects cannot be explained by those T-odd effects arising from hard scattering and are related to the non-perturbative nature of the hadrons. Indeed, these T-odd effects can be generated from final or initial interactions between active partons involved in the hard scattering and remnant partons in hadrons [15,23,24]. The effect of these interactions can be represented by gauge links in the definitions of the parton distributions [23,24]. Theoretically there are two approaches to explain single spin azimuthal asymmetry by taking non-perturbative nature of hadrons into account. One is to take the transverse momenta k_T of the partons in a hadron into account where one uses transverse momentum dependent parton distributions to parameterize non-perturbative effects. For a polarized hadron as an initial state the effect is parameterized by a Sivers function [6], while for a hadron observed in a final state the T-odd effect related to this hadron is parameterized by a Collins function [7]. For semi-inclusive deep-inelastic scattering, both functions can make contributions to the observed single spin azimuthal asymmetry. Single spin azimuthal asymmetry has been studied in terms of these functions [8–13]. These functions have been also studied with models [14–17]. Another approach, called the Qiu–Sterman mechanism, is that the T-odd effect is produced by taking twist-3 effect into account, it being proportional to quark–gluon correlations inside a hadron [18]. The fact that T-odd effects can be generated at twist-3 level was also pointed out in [19]. This approach was used to make predictions for various processes in [18,20]. It is interesting to note that at first sight the physical reason for the effect is different in different approaches. In the first approach the helicity of a initial hadron is changed because of orbital angular momenta of the partons. This can be seen clearly in terms of light-cone wave functions [21]. In the second approach the helicity flip is caused by the non-zero spin of the gluon which is correlated with other partons. Predictions based

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on different approaches are different. A question arises as to why there are two physical origins for one effect.

This question has been answered partly by recent studies of transverse momentum dependent parton distributions [22–24], which are involved in the first approach. It has been shown that gauge links in these distributions play an important role to incorporate T-odd effects introduced by final state interactions. In particular, additional gauge links should be included in the definitions of these distributions [24]. With these gauge links it is possible to relate the second k_{T} moment of the Sivers function to the twist-3 matrix element in the second approach [25]. With relations between the non-perturbative matrix elements in the different approaches it may be possible to unify the two approaches and to have unique predictions for the single spin azimuthal asymmetries. In this letter we will show that predictions based on the two approaches are still different, although such relations exist. We will show this in detail with the Drell–Yan process. Before showing this we give another derivation of the relations between the second k_{T} moments of T-odd distributions and twist-3 matrix elements by using the time-reversal symmetry of QCD.

We consider a proton moving in the z-direction with the momentum P and the transverse spin $\mathbf{s}_{\mathrm{T}}$. We use a light-cone coordinate system and introduce two light-cone vectors: $n^\mu = (0,1,0,0)$, $l^\mu = (1,0,0,0)$ and $n \cdot l = 1$. Neglecting the proton mass we have $P^\mu = (P^+,0,0,0)$. Taking transverse momenta of partons in the quark–quark correlation in a proton into account, there are two T-odd parton distribution functions appearing in a Drell–Yan process which take the effects of the initial state interaction into account. They can be defined as [26]

$$
\begin{aligned}
&f^{\perp}_{1\mathrm{T,DY}}(x,k^2_{\mathrm{T}})\varepsilon_{\perp\mu\nu}k^\mu_{\mathrm{T}}s^\nu_{\mathrm{T}} \\
&\quad = \frac{1}{4}\int \frac{\mathrm{d}\xi^-\mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3}\mathrm{e}^{-\mathrm{i}k\cdot\xi}\left\{\langle P,\mathbf{s}_{\mathrm{T}}|\bar\psi(\xi)\gamma^+V(\xi)\psi(0)|P,\mathbf{s}_{\mathrm{T}}\rangle \right.\\
&\qquad \left. -(\mathbf{s}_{\mathrm{T}}\to -\mathbf{s}_{\mathrm{T}})\right\},\\
&h^{\perp}_{1,\mathrm{DY}}(x,k^2_{\mathrm{T}})k^i_{\mathrm{T}} \\
&\quad = -\int \frac{\mathrm{d}\xi^-\mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3}\mathrm{e}^{-\mathrm{i}k\cdot\xi}\cdot\langle P|\bar\psi(\xi)\sigma^{+i}V(\xi)\psi(0)|P\rangle, \qquad (1)
\end{aligned}
$$

with $\xi^\mu = (0,\xi^-,\xi_{\mathrm{T}})$ and $\varepsilon_{\perp\mu\nu} = \varepsilon_{\rho\sigma\mu\nu}n^\rho l^\sigma$. The momentum k is $k^\mu = (xP^+,0,\mathbf{k}_{\mathrm{T}})$. The matrix element in the last line is spin averaged. The function $f^{\perp}_{1\mathrm{T,DY}}(x,k^2_{\mathrm{T}})$ is the Sivers function for Drell–Yan processes. $V(\xi)$ is a product of gauge links to make the matrix element gauge invariant; it takes the effect of the initial state interaction in the Drell–Yan process into account. If one can take $V(\xi)$ as a unit matrix, then one can show with time-reversal symmetry that both correlation functions are zero. It is important to note that $V(\xi)$ is not a unit matrix; even in the light-cone gauge $n\cdot G=0$, additional gauge links must be introduced to make the definitions gauge invariant [24]. We will take the light-cone gauge. In this gauge $V(\xi)$ reads

$$
\begin{aligned}
V(\xi) &= V_{-\infty}(\xi_{\mathrm{T}}) \\
&= P\exp\left(\mathrm{i}g\int_0^{\xi_{\mathrm{T}}}\mathrm{d}\xi_{\mathrm{T}}\cdot\mathbf{G}_{\mathrm{T}}(0,\xi^-=-\infty,\xi_{\mathrm{T}})\right). \qquad (2)
\end{aligned}
$$

This gauge link takes the effects of the initial state interaction into account and it can be derived in a similar way as in SIDIS [24]. The difference is that the gauge link is at $\xi^- = -\infty$ because it is for the initial state interaction.

Under parity- and time-reversal transformation, we obtain for the matrix element

$$
\begin{aligned}
&\langle P,\mathbf{s}_{\mathrm{T}}|\bar\psi(\xi)\gamma^+V_{-\infty}(\xi_{\mathrm{T}})\psi(0)|P,\mathbf{s}_{\mathrm{T}}\rangle \\
&\quad = \langle P,-\mathbf{s}_{\mathrm{T}}|\bar\psi(\xi)\gamma^+V_{\infty}(\xi_{\mathrm{T}})\psi(0)|P,-\mathbf{s}_{\mathrm{T}}\rangle, \qquad (3)
\end{aligned}
$$

with

$$
V_{\infty}(\xi_{\mathrm{T}}) = P\exp\left(\mathrm{i}g\int_0^{\xi_{\mathrm{T}}}\mathrm{d}\xi_{\mathrm{T}}\cdot\mathbf{G}_{\mathrm{T}}(0,\xi^-=\infty,\xi_{\mathrm{T}})\right). \qquad (4)
$$

Similarly one can define the two T-odd parton distribution functions appearing in deep-inelastic processes which take the effects of the final state interaction into account. The two functions $f^{\perp}_{1\mathrm{T,DIS}}(x,k^2_{\mathrm{T}})$ and $h^{\perp}_{1,\mathrm{DIS}}(x,k^2_{\mathrm{T}})$ are defined similarly as in (1), but with the gauge link $V(\xi)$ replaced with $V_\infty(\xi_{\mathrm{T}})$. These two functions are related to those in Drell–Yan processes with time-reversal symmetry. With (3) we can write

$$
\begin{aligned}
&f^{\perp}_{1\mathrm{T,DY}}(x,k^2_{\mathrm{T}})\varepsilon_{\perp\mu\nu}k^\mu_{\mathrm{T}}s^\nu_{\mathrm{T}} = \frac{1}{8}\int\frac{\mathrm{d}\xi^-\mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3}\mathrm{e}^{-\mathrm{i}k\cdot\xi} \\
&\quad \times\left\{\langle P,\mathbf{s}_{\mathrm{T}}|\bar\psi(\xi)\gamma^+\left[V_{-\infty}(\xi_{\mathrm{T}})-V_{\infty}(\xi_{\mathrm{T}})\right]\psi(0)|P,\mathbf{s}_{\mathrm{T}}\rangle\right.\\
&\quad \left. -(\mathbf{s}_{\mathrm{T}}\to-\mathbf{s}_{\mathrm{T}})\right\},\\
&h^{\perp}_{1,\mathrm{DY}}(x,k^2_{\mathrm{T}})k^i_{\mathrm{T}} = -\frac{1}{2}\int\frac{\mathrm{d}\xi^-\mathrm{d}^2\xi_{\mathrm{T}}}{(2\pi)^3}\mathrm{e}^{-\mathrm{i}k\cdot\xi}\\
&\quad \times\langle P|\bar\psi(\xi)\sigma^{+i}\left[V_{-\infty}(\xi_{\mathrm{T}})-V_{\infty}(\xi_{\mathrm{T}})\right]\psi(0)|P\rangle. \qquad (5)
\end{aligned}
$$

It is expected that the functions $f^{\perp}_{1\mathrm{T,DY}}$ and $h^{\perp}_{1,\mathrm{DY}}$ decrease rapidly with increasing k_{T}. Then T-odd effects related to them can be estimated at leading order by the second moment of k_{T} of the left hand side in (5):

$$
\begin{aligned}
K^\alpha_f(x) &= \int\mathrm{d}^2k_{\mathrm{T}}k^\alpha_{\mathrm{T}}f^{\perp}_{1\mathrm{T,DY}}(x,k^2_{\mathrm{T}})\varepsilon_{\perp\mu\nu}k^\mu_{\mathrm{T}}s^\nu_{\mathrm{T}}\\
&= -\frac{1}{2}\varepsilon^{\alpha\sigma}s_{T\sigma}\int\mathrm{d}^2k_{\mathrm{T}}|\mathbf{k}_{\mathrm{T}}|^2f^{\perp}_{1\mathrm{T,DY}}(x,k^2_{\mathrm{T}}),\\
K^{\mu\nu}_h(x) &= \int\mathrm{d}^2k_{\mathrm{T}}k^\mu_{\mathrm{T}}h^{\perp}_{1,\mathrm{DY}}(x,k^2_{\mathrm{T}})k^\nu_{\mathrm{T}} \qquad (6)\\
&= \frac{1}{2}(n^\mu l^\nu+n^\nu l^\mu-g^{\mu\nu})\int\mathrm{d}^2k_{\mathrm{T}}|\mathbf{k}_{\mathrm{T}}|^2h^{\perp}_{1,\mathrm{DY}}(x,k^2_{\mathrm{T}}).
\end{aligned}
$$

With (5) these moments can be expressed in terms of matrix elements. Taking K^α_f as an example, we have

$$
\begin{aligned}
K^\alpha_f(x) &= -\frac{1}{8}\int\frac{\mathrm{d}\xi^-}{(2\pi)}\mathrm{e}^{-\mathrm{i}xP^+\xi^-}\times\mathrm{i}\frac{\partial}{\partial\xi_{T\alpha}}\\
&\quad\times\left\{\langle P,\mathbf{s}_{\mathrm{T}}|\bar\psi(\xi)\gamma^+\left[V_{-\infty}(\xi_{\mathrm{T}})-V_{\infty}(\xi_{\mathrm{T}})\right]\psi(0)|P,\mathbf{s}_{\mathrm{T}}\rangle\right.\\
&\quad\left.-(\mathbf{s}_{\mathrm{T}}\to-\mathbf{s}_{\mathrm{T}})\right\}|_{\xi_{\mathrm{T}}=0}. \qquad (7)
\end{aligned}
$$

Taking the derivatives we have

$$\begin{aligned}K_f^\alpha(x) &= \frac{1}{8}\int \frac{\mathrm{d}\xi^-}{(2\pi)}\mathrm{e}^{-\mathrm{i}xP^+\xi^-} \\ &\times \left\{ g\langle P,\mathbf{s}_\mathrm{T}|\bar\psi(\xi^- n)\gamma^+ \right. \\ &\quad\times \left[G^\alpha_\mathrm{T}(0,\infty,0,0) - G^\alpha_\mathrm{T}(0,-\infty,0,0)\right]\psi(0)|P,\mathbf{s}_\mathrm{T}\rangle \\ &\quad \left. -(\mathbf{s}_\mathrm{T}\to -\mathbf{s}_\mathrm{T})\right\}. \end{aligned} \tag{8}$$

Now one can show that K^α is related to the twist-3 quark gluon correlation $T_F(x,x)$ introduced in [18]. The correlation function is defined as

$$\begin{aligned}&T_F(x_1,x_2)\epsilon^{\mu\nu\sigma\rho}n_\nu l_\sigma s_{T\rho} \\ &= -\frac{g}{2}\int \frac{\mathrm{d}y_1\mathrm{d}y_2}{4\pi}\mathrm{e}^{-\mathrm{i}y_2(x_2-x_1)P^+ - \mathrm{i}y_1x_1P^+} \\ &\quad\times \left\{\langle P,\mathbf{s}_\mathrm{T}|\bar\psi(y_1 n)\gamma^+ G^{+\mu}(y_2n)\psi(0)|P,\mathbf{s}_\mathrm{T}\rangle \right.\\ &\quad \left. -(\mathbf{s}_\mathrm{T}\to -\mathbf{s}_\mathrm{T})\right\}, \end{aligned} \tag{9}$$

where we include the coupling constant g into the definition. It is straightforward to obtain

$$T_F(x,x) = \int \mathrm{d}^2k_\mathrm{T}|\mathbf{k}_\mathrm{T}|^2 f^\perp_{1\mathrm{T},\mathrm{DY}}(x,k_\mathrm{T}^2). \tag{10}$$

Similarly we have

$$T_H(x,x) = \int \mathrm{d}^2k_\mathrm{T}|\mathbf{k}_\mathrm{T}|^2 h^\perp_{1,\mathrm{DY}}(x,k_\mathrm{T}^2), \tag{11}$$

where T_H is defined with a twist-3 operator:

$$\begin{aligned}T_H(x_1,x_2) &= g\int \frac{\mathrm{d}y_1\mathrm{d}y_2}{4\pi}\mathrm{e}^{-\mathrm{i}y_2(x_2-x_1)P^+ - \mathrm{i}y_1x_1P^+} \\ &\quad\times \langle P|\bar\psi(y_1n)\sigma^{+\mu}G^+_{\ \mu}(y_2n)\psi(0)|P\rangle. \end{aligned} \tag{12}$$

The relations in (10) and (11) clearly show that the effect of the orbital angular momenta of the quarks is closely related to that of the quark–gluon correlations because of gauge invariance. These relations also show that the non-perturbative effects in the two approaches for the single spin azimuthal asymmetries are the same. However, it should be noted that perturbative coefficients in these two approaches are calculated in different ways. In the first approach one uses k_T factorization, while a collinear expansion is used in the second approach. If the perturbative coefficients in the two approaches are related in a consistent way so that the predicted single spin azimuthal asymmetries are the same, then we may have a unique prediction for single spin azimuthal asymmetries and the question asked before is fully answered. It is difficult to establish a general relation between the perturbative coefficients, since they are differently calculated in different ways and are different in different processes. But we can show that the single spin azimuthal asymmetry in the Drell–Yan process is *differently* predicted by the two approaches.

Now we calculate the single spin asymmetry in the Drell–Yan process:

$$A(P_A,\mathbf{s}_\mathrm{T}) + B(P_B) \to l^-(P_1) + l^+(P_2) + X, \tag{13}$$

where the proton A is transversely polarized with the spin vector $\mathbf{s}_\mathrm{T}$ and moves in the $+z$-direction. The x-direction is chosen as the direction of $\mathbf{s}_\mathrm{T}$. We have $S = (P_A+P_B)^2$. The hadron B is unpolarized and moves in the $-z$-direction. We will calculate the single spin azimuthal asymmetry at leading orders, where the lepton pair has a small transverse momentum. We assume that the solid angle $\Omega(\theta,\phi)$ of the produced lepton in the center-of-mass frame of the produced lepton pair and the invariant mass Q^2 of the lepton pair is observed. The single spin asymmetry is defined as

$$A_N = \left(\frac{\mathrm{d}\sigma(\mathbf{S}_\mathrm{T})}{\mathrm{d}Q^2\mathrm{d}\Omega} - \frac{\mathrm{d}\sigma(-\mathbf{S}_\mathrm{T})}{\mathrm{d}Q^2\mathrm{d}\Omega}\right)\Big/\left(\frac{\mathrm{d}\sigma(\mathbf{S}_\mathrm{T})}{\mathrm{d}Q^2\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma(-\mathbf{S}_\mathrm{T})}{\mathrm{d}Q^2\mathrm{d}\Omega}\right). \tag{14}$$

The asymmetry is calculated in [27] with the Qiu–Sterman mechanism. The result reads

$$\begin{aligned}A_N &= -\frac{1}{Q}\cdot\frac{\sin 2\theta\sin\phi}{1+\cos^2\theta} \\ &\times \left(\frac{\sum_q e_q^2\int \mathrm{d}x_A\mathrm{d}x_B\delta(Q^2-x_Ax_BS)T_{F,q/A}(x_A,x_A)f_{\bar q/B}(x_B)}{\sum_q e_q^2\int \mathrm{d}x_A\mathrm{d}x_B f_{q/A}(x_A)f_{\bar q/B}(x_B)} \right.\\ &\qquad \left. + \cdots\right), \end{aligned} \tag{15}$$

where $T_{F,q/P}(x,x)$ is defined in (9). The subscript q/A denotes the distribution of q in hadron A. It should be noted that the summation $\sum_q$, also below, is over all quark and antiquark flavors, i.e., q can be an antiquark in the summation. We only keep the term with $T_{F,q/A}$. The dots represent another term proportional to $T_{H,\bar q/B}$ which is irrelevant in this letter.

In order to make a comparison of the two approaches we study the single spin asymmetries with k_T dependent distributions. At tree level the partonic process is just $q\bar q \to l^+l^-$. The cross section can be written

$$\sigma = e^2\frac{1}{2S}\int \frac{\mathrm{d}^3P_1}{(2\pi)^3 2P_1^0}\frac{\mathrm{d}^3P_2}{(2\pi)^3 2P_2^0}L_{\mu\nu}W^{\mu\nu}\cdot\frac{1}{Q^4}, \tag{16}$$

where the leptonic tensor $L_{\mu\nu}$ and the hadronic tensor $W^{\mu\nu}$ are given by

$$\begin{aligned}L^{\mu\nu} &= 4(P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - P_1\cdot P_2 g^{\mu\nu}), \\ W^{\mu\nu} &= \sum_q e_q^2\int\frac{\mathrm{d}^4k_A}{(2\pi)^4}\frac{\mathrm{d}^4k_B}{(2\pi)^4} \\ &\times (2\pi)^4\delta^4(Q-k_A-k_B) \\ &\times \int \mathrm{d}^4\xi_1\mathrm{d}^4\xi_2\mathrm{e}^{\mathrm{i}k_A\cdot\xi_1+\mathrm{i}k_B\cdot\xi_2}\cdot(\gamma^\nu)_{jk}\cdot(\gamma^\mu)_{li} \\ &\times \left[\langle P_A,s_\mathrm{T}|\bar q_j(0)q_i(\xi_1)|P_A,s_\mathrm{T}\rangle\langle P_B|q_k(0)\bar q_l(\xi_2)|P_B\rangle \right.\\ &\qquad \left. +\cdots\right], \end{aligned} \tag{17}$$

where $\cdots$ denotes power-suppressed terms. The quark q and $\bar q$ carries the momentum

$$\begin{aligned}k_A &= x_AP_A + k_{A\mathrm{T}}, \\ k_B &= x_BP_B + k_{B\mathrm{T}}, \end{aligned} \tag{18}$$

respectively. The quark density matrix with k_T dependence

$$\begin{aligned}&\Phi_{ij}(x,k_\mathrm{T};P,S)\\&=\frac{1}{2}\int\frac{\mathrm{d}\xi^-\mathrm{d}^2\xi_\mathrm{T}}{(2\pi)^3}\mathrm{e}^{\mathrm{i}k\cdot\xi}\langle P,S|\bar{\psi}_j(0)\psi_i(\xi)|P,S\rangle|_{\xi^+=0}\end{aligned}\tag{19}$$

can be parameterized as [26]

$$\begin{aligned}&\Phi(x,k_\mathrm{T};P,S)\\&=\frac{1}{4}\left\{f_1\not{l}+f^{\perp}_{1\mathrm{T,DY}}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu l^\nu k^\rho_\mathrm{T}s^\sigma_\mathrm{T}+g_{1s}\gamma_5\not{l}\right.\\&\quad\left.+h_{1\mathrm{T}}\mathrm{i}\sigma_{\mu\nu}\gamma_5 l^\mu s^\nu_\mathrm{T}+h^{\perp}_{1s}\mathrm{i}\sigma_{\mu\nu}\gamma_5 l^\mu k^\nu_\mathrm{T}+h^{\perp}_{1,\mathrm{DY}}\sigma_{\mu\nu}k^\mu_\mathrm{T}l^\nu\right\},\end{aligned}\tag{20}$$

where the Sivers function is $f^{\perp}_{1\mathrm{T,DY}}$. The functions $f^{\perp}_{1\mathrm{T,DY}}$ and $h^{\perp}_{1,\mathrm{DY}}$ are defined in (1). We changed the notation of [26] slightly by replacing $1/M$ with 1. With this parameterization we have

$$\begin{aligned}W^{\mu\nu}&=\frac{1}{3}\sum_q e_q^2\int\mathrm{d}k^+_A\mathrm{d}^2k_{A\mathrm{T}}\mathrm{d}k^-_B\mathrm{d}^2k_{B\mathrm{T}}\\&\quad\times(2\pi)^4\delta^4(Q-k_A-k_B)(\mathbf{k}_{A\mathrm{T}}\times\mathbf{s}_\mathrm{T})\cdot\hat{z}\\&\quad\times f^{\perp}_{1\mathrm{T},q/A}(k_A)f_{1,\bar{q}/B}(k_B)[g^{\mu\nu}-l^\nu n^\mu-l^\mu n^\nu]+\cdots,\end{aligned}\tag{21}$$

where we keep only terms with $f^{\perp}_{1\mathrm{T,DY}}$. $\hat{z}$ denotes the direction of the z-axis. It should be noted that the total momentum Q of the lepton pair has non-zero transverse components in general. It depends on the transverse momenta of incoming partons. It is now straightforward to calculate the asymmetry defined in (14). Since the asymmetry is defined as a distribution of variables in the center-of-mass frame of the lepton pair, we need to specify the frame. We assume that the center-of-mass frame is obtained from the laboratory frame by a Lorentz boost only. This is conveniently used in experiment. In the center-of-mass frame the lepton l^+ and l^- has the momentum k_2 and k_1 respectively. The momentum k_1 and k_2 read

$$\begin{aligned}k_1^\mu&=\frac{\sqrt{Q^2}}{2}(1,\sin\theta\sin\phi,\sin\theta\cos\phi,\cos\phi),\\k_2^\mu&=\frac{\sqrt{Q^2}}{2}(1,-\sin\theta\sin\phi,-\sin\theta\cos\phi,-\cos\phi).\end{aligned}\tag{22}$$

The momentum P_i ($i=1,2$) in the laboratory frame is related to k_i ($i=1,2$) by the boost

$$\begin{aligned}P_i^0&=\frac{Q^0}{\sqrt{Q^2}}\left(k_i^0+\frac{\mathbf{Q}\cdot\mathbf{k}_i}{Q^0}\right),\\\mathbf{P}_i&=\mathbf{k}_i+\frac{k_i^0}{\sqrt{Q^2}}\mathbf{Q}+\left(\frac{Q^0}{\sqrt{Q^2}}-1\right)\frac{\mathbf{Q}\cdot\mathbf{k}_i}{\mathbf{Q}\cdot\mathbf{Q}}\mathbf{Q}.\end{aligned}\tag{23}$$

The phase space integration is invariant under the boost. Using (23) one can express $L^{\mu\nu}$ in terms of k_1, k_2 and Q. For transverse momentum independent parton distributions one expects in general that they decrease rapidly with increasing transverse momenta. Hence an expansion of the perturbative part in transverse momenta is a good approximation. Keeping the first non-zero order in the expansion of $k_{A\mathrm{T}}$ and $k_{B\mathrm{T}}$, we obtain the asymmetry

$$\begin{aligned}A_N&=\frac{1}{Q}\cdot\frac{\sin 2\theta\sin\phi}{1+\cos^2\theta}\\&\quad\times\sum_q e_q^2\int\mathrm{d}x_A\mathrm{d}x_B\delta(Q^2-x_Ax_BS)\frac{x_B-x_A}{2(\sqrt{x_A}+\sqrt{x_B})^2}\\&\quad\times f_{\bar{q}/B}(x_B)\cdot\int\mathrm{d}^2k_\mathrm{T}|\mathbf{k}_\mathrm{T}|^2f^{\perp}_{1\mathrm{T,DY},q/A}(x_A,k^2_\mathrm{T})\\&\quad\Big/\sum_q e_q^2\int\mathrm{d}x_A\mathrm{d}x_Bf_{q/A}(x_A)f_{\bar{q}/B}(x_B)+\cdots\\&=-\frac{1}{Q}\cdot\frac{\sin 2\theta\sin\phi}{1+\cos^2\theta}\\&\quad\times\sum_q e_q^2\int\mathrm{d}x_A\mathrm{d}x_B\delta(Q^2-x_Ax_BS)\frac{x_A-x_B}{2(\sqrt{x_A}+\sqrt{x_B})^2}\\&\quad\times f_{\bar{q}/B}(x_B)\cdot T_{F,q/P}(x_A,x_A)\\&\quad\Big/\sum_q e_q^2\int\mathrm{d}x_A\mathrm{d}x_Bf_{q/A}(x_A)f_{\bar{q}/B}(x_B)+\cdots\end{aligned}\tag{24}$$

In the above equation we have assumed that the initial hadrons are in a center-of-mass frame, i.e., $P^0_A=P^0_B$. In the last step we have used the relation in (10). Again, the summation $\sum_q$ is over all quark and antiquark flavors. It is clear that the asymmetry here is different from that in (15) because of the factor $(x_A-x_B)/2(\sqrt{x_A}+\sqrt{x_B})^2$. If the factor was 1, then the asymmetry would be the same. Hence, the asymmetries obtained by the two approaches will have the same angular distribution but the normalization is different. Since the factor can be positive or negative, the asymmetries from the two approaches can even have different signs.

It should be noted that the hadronic tensor calculated with the parameterization in (20) is not invariant under electromagnetic gauge transformation. This can be seen by evaluating $Q_\mu W^{\mu\nu}$ with $W^{\mu\nu}$ given in (21). The reason is that the partons involved in the hard scattering have non-zero transverse momenta and $\gamma\cdot l=\gamma^-$ is contracted with the hard part of $q\bar{q}\to l^+l^-$ according to the first two terms in (20). With non-zero transverse momenta the contraction with γ^- does not make the $q\bar{q}$ pair on-shell. Hence the $U_\mathrm{em}(1)$ gauge invariance is not preserved. It is also pointed out [28] that the decomposition in (20) may need to be reexamined because the density matrix element also depends on the vector n^μ due to gauge links. This dependence is neglected in (20). There are possibly many ways to restore the gauge invariance. In this letter we simply make the initial parton on-shell by replacing γ^- with $\gamma\cdot k/n\cdot k$ for the first two terms in (20), i.e.,

$$\begin{aligned}&\Phi(x,k_\mathrm{T};P,s_\mathrm{T})\\&=\frac{1}{4}\left\{f_1(x,k^2_\mathrm{T})\frac{\gamma\cdot k}{k\cdot n}+f^{\perp}_{1\mathrm{T,DY}}(x,k^2_\mathrm{T})\frac{\gamma\cdot k}{k\cdot n}\varepsilon_{\perp\rho\sigma}k^\rho_\mathrm{T}s^\sigma_\mathrm{T}\right.\\&\quad\left.+\cdots\right\}.\end{aligned}\tag{25}$$

The hadronic tensor obtained with (25) is $U_{\rm em}(1)$ gauge invariant. It is

$$W^{\mu\nu} = \frac{1}{3}\sum_q e_q^2 \int \mathrm{d}k_A^+ \mathrm{d}^2 k_{A\mathrm{T}} \mathrm{d}k_B^- \mathrm{d}^2 k_{B\mathrm{T}} \times (2\pi)^4 \delta^4(Q - k_A - k_B)(\mathbf{k}_{A\mathrm{T}} \times \mathbf{s}_\mathrm{T}) \cdot \hat{z} \times \frac{1}{(k_A \cdot n)(k_B \cdot l)} \times \left[g^{\mu\nu} k_A \cdot k_B - k_A^\mu k_B^\nu - k_A^\nu k_B^\mu\right] f^\perp_{1\mathrm{T},\mathrm{DY},q/A}(k_A) f_{1,\bar{q}/B}(k_B). \quad (26)$$

It is straightforward to show that $Q_\mu W^{\mu\nu} \sim k_A^2 k_B^\nu + k_B^2 k_A^\nu \sim k_\mathrm{T}^2$. Hence the gauge invariance is preserved up to order k_T^2. The asymmetry calculated with this tensor will be gauge invariant. The result of A_N can be obtained from (24) by replacing the factor $(x_B - x_A)/2(\sqrt{x_A} + \sqrt{x_B})^2$ with $\sqrt{x_B}/(\sqrt{x_A} + \sqrt{x_B})$. Therefore, even after we make the hadronic tensor gauge invariant, the obtained asymmetry A_N is still different from that in (15) from the second approach. It is interesting to see how the same asymmetry in (15) can be obtained by starting from (26). If we replace the tensor $[g^{\mu\nu} k_A \cdot k_B - k_A^\mu k_B^\nu - k_A^\nu k_B^\mu]$ with $[g^{\mu\nu} k_A \cdot k'_B - k_A^\mu k_B'^\nu - k_A^\nu k_B'^\mu]$, where $k_B'^\mu = (0, k_B^-, -\mathbf{k}_{A\mathrm{T}})$, and we neglect the dependence of the lepton momenta on the transverse momenta of the incoming partons, we indeed obtain the asymmetry in (15) with the same normalization, but with an extra negative sign. However, the transverse momentum of the lepton pair cannot be neglected and we cannot do the replacement.

To summarize: There are two different approaches for single spin azimuthal asymmetries. Using time-reversal symmetry, we give in this letter a detailed derivation of the relations between k_T dependent T-odd distributions and twist-3 quark–gluon correlators, which are used in different approaches, respectively. These relations show that the physical origin in the two different approaches for single spin azimuthal asymmetries is the same because of the gauge invariance. With these relations it may be expected that one can unify these two approaches and is able to deliver a unique prediction for single spin azimuthal asymmetries. We have studied in detail the single spin azimuthal asymmetry in the Drell–Yan process with the Sivers function and found that predictions from different approaches are different even with these relations. The k_T factorization used for single spin azimuthal asymmetries does not respect the $U_{\rm em}(1)$ gauge invariance. This problem may be solved by changing the projection of the perturbative part slightly. But even after this change the predicted asymmetry is still different. Our study shows clearly that different approaches give different predictions for the same physical effect in the Drell–Yan process and one can expect that the same situation will also appear in other processes. Therefore, at present we have not a unique prediction for single spin azimuthal asymmetries, in the Drell–Yan process at least, and this problem needs to be studied further.

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